The Chemistry of Calculus: The Basic Formulas and Rules for Differentiation

Structure of Functions

Whenever you are faced with a function, determine its structure! Is it a sum, a product, a quotient, or a function inside a function? What parts are constants, either additive constants or multiplicative constants? Often it is helpful to put a function into a more ‘power-full’ form: $1/x^2 = x^{-2}$, $\sqrt{1 - x^2} = (1 - x^2)^{1/2}$.

‘ATOMS’: Basic Formulas for Differentiation

Additive Constants: $\frac{d}{dx}(c) = 0$

Powers: $\frac{d}{dx}(x^n) = nx^{n-1}$ Note: powers have variable in bottom, constant in top.

Exponentials: $\frac{d}{dx}(e^x) = e^x$ Self-replicating! Note: exponentials have variable in exponent and constant in bottom.

Logarithms: $\frac{d}{dx} \ln(x) = \frac{1}{x} = x^{-1}$

Trig functions: $\frac{d}{dx} \sin(x) = \cos(x), \frac{d}{dx} \cos(x) = -\sin(x)$

‘Molecular Bonds’: Basic Rules for Differentiation

Multiplicative Constant Rule: $\frac{d}{dx}(c \cdot f(x)) = c \frac{d}{dx} f(x)$ (Constants are your friends!)

Sum Rule: $(f + g)' = f' + g'$ The derivative of a sum is the sum of the derivatives.

Product Rule: $(f \cdot g)' = f' \cdot g + f \cdot g'$

Quotient Rule:

$\left( \frac{f}{g} \right)' = \frac{f'g - fg'}{g^2}$

Chain Rule: ‘a function inside a function’

$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$

The derivative of the outside function times the derivative of the inside.

Here $u$ is the inside function: the gUts of the function, the Unified inside.

Special Cases that Students often Memorize:

\[
\frac{d(x^2)}{dx} = 2x, \quad \frac{d(e^{ax})}{dx} = a e^{ax}, \quad \frac{d(b^x)}{dx} = \ln(b) b^x
\]
The Chemistry of Calculus: The Basic Formulas and Rules for Integration

Structure of Functions

When integrating, you should determine the structure of the function being integrated. Is it a sum or is there a function inside a function? What parts are constants, either additive constants or multiplicative constants? If it is a product or quotient, sometimes algebra can transform the function. Often it is helpful to put a function into a more ‘power-full’ form: \(1/(2x+3)^2 = (2x+3)^{-2}, \sqrt{1-x^2} = (1-x^2)^{1/2}\).

‘ATOMS’: Basic Formulas for Integration

Antiderivative: \(\int dx = x + C\)

Powers: \(\int x^n \, dx = \frac{x^{n+1}}{n+1} + C\) if \(n \neq -1\).

When \(n = -1\): \(\int \frac{1}{x} \, dx = \ln(x) + C\)

Exponentials: \(\int e^x \, dx = e^x + C\) Self-replicating!

Trig functions: \(\int \sin(x) \, dx = -\cos(x) + C, \int \cos(x) \, dx = \sin(x) + C\)

‘Molecular Bonds’: Basic Rules for Integration

Multiplicative Constant Rule: \(\int c \cdot f(x) \, dx = c \int f(x) \, dx\) (Constants are your friends!)

Sum Rule: \(\int (f + g) = \int f + \int g\) The integral of a sum is the sum of the integrals.

U-Substitution—the chain rule backwards: Choose ‘u’ to be the ”united” inside function (the gUts inside a function). Calculate ‘du’ and substitute everywhere, including the limits.

Integration by Parts—the product rule backwards:

\[
\int_a^b u \, dv = uv \bigg|_a^b - \int_a^b v \, du
\]

Special Algebra Techniques to Transform Integrand: partial fraction decomposition, trig substitution, complex exponentials, etc.

Special Cases that Students often Memorize:

\(\int x \, dx = x^2/2 + C, \int x^2 \, dx = x^3/3 + C, \int e^{ax} \, dx = e^{ax}/a + C\)