The Chemistry of Calculus: The Basic Formulas and Rules for Differentiation

Structure of Functions

Whenever you are faced with a function, determine its structure! Is it a sum, a product, a quotient, or a function inside a function? What parts are constants, either additive constants or multiplicative constants? Often it is helpful to put a function into a more ‘power-full’ form: \( 1 = x^2 = x^2 \), \( \frac{1}{x^2} = (1 \cdot x^2)^{-2} \).

`ATOM'S': Basic Formulas for Differentiation

Additive Constants: \( \frac{d}{dx}(c) = 0 \)

Powers: \( \frac{d}{dx}(x^n) = nx^{n-1} \)  
Note: powers have variable in bottom, constant in top.

Exponentials: \( \frac{d}{dx}(e^x) = e^x \)  
Self-replicating! Note: exponentials have variable in exponent and constant in bottom.

Logarithms: \( \frac{d}{dx}\ln(x) = \frac{1}{x} = x^{-1} \)

Trig functions: \( \frac{d}{dx}\sin(x) = \cos(x), \frac{d}{dx}\cos(x) = -\sin(x) \)

`Molecular Bonds': Basic Rules for Differentiation

Multiplicative Constant Rule: \( \frac{d}{dx}(cf(x)) = c\frac{d}{dx}f(x) \)  
Constants are your friends!

Sum Rule: \( (f + g)' = f' + g' \)  
The derivative of a sum is the sum of the derivatives.

Product Rule: \( (f \cdot g)' = f' \cdot g + f \cdot g' \)

Quotient Rule: \( \frac{df}{g} = \frac{f'g - fg'}{g^2} \)

Chain Rule: "a function inside a function"

\[ \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \]

Here \( u \) is the inside function: the gUts of the function, the Uni-ed inside.
The Chemistry of Calculus: The Basic Formulas and Rules for Integration

Structure of Functions

When integrating, you should determine the structure of the function being integrated. Is it a sum or is there a function inside a function? What parts are constants, either additive constants or multiplicative constants? If it is a product or quotient, sometimes algebra can transform the function. Often it is helpful to put a function into a more 'power-full' form: \(1=(2x+3)^2 = (2x+3)^{1/2}\), \(\frac{1}{x^2} = (1-x^2)^{1/2}\).

`ATOMS': Basic Formulas for Integration

Antiderivative: \(\int dx = x + C\)

Powers: \(\int x^n \, dx = \frac{x^{n+1}}{n+1} + C \quad \text{if } n \neq 1\).

When \(n = 1\): \(\int 1 \, dx = \ln(x) + C\)

Exponentials: \(\int e^x \, dx = e^x + C\) Self-replicating!

Trig functions: \(\int \sin(x) \, dx = -\cos(x) + C\), \(\int \cos(x) \, dx = \sin(x) + C\)

`Molecular Bonds': Basic Rules for Integration

Multiplicative Constant Rule: \(\int cf(x) \, dx = cf(x) \, dx \quad \text{Constants are your friends!}\)

Sum Rule: \(\int (f + g) \, dx = \int f \, dx + \int g \, dx \quad \text{The integral of a sum is the sum of the integrals.}\)

U-Substitution| the chain rule backwards: Choose 'u' to be the inside function (the gUts of a function) of some part of the integrand. Calculate 'du' and substitute everywhere, including the limits.

Integration by Parts| the product rule backwards:
\[
\int_a^b uv \, dx = uv \big|_a^b - \int_a^b u \, dv
\]

Special Algebra Techniques to Transform Integrand: partial fraction decomposition, trig substitution, complex exponentials, etc.