

Senior Seminar Ideas and Advice

Concepts to stress in developing ideas:

Ask mathematical questions such as

“Is there *any* upper or lower bound I can prove or conjecture?”

“Can I change the problem to make it easier to solve or easier to see a pattern?”

“If I see no pattern, can I use statistics to support a conjecture assuming something is ‘random’?”

“Do I believe this proof? Or do I need to include more details for my audience?”

Some technical details:

Use math reference style, not English footnote style.

Use [] not () for references and especially avoid superscript notation for notes.

State theorems and lemmas and definitions and conjectures formally on separate lines to make it easier for a reader to skim the paper and to see its organization.

Make sure definitions are precise, and always give one or two examples.

References:

Unlike English and History professors, mathematicians are lazier and more vain! At least the usual math reference style seems to indicate this. We use far fewer references than in the humanities, and mostly in the intro sections where one is attributing results, that is, saying who did what.

Example:

“... The result that N is finite follows easily from an effective result of Ellison [El], which is a response to an earlier result of Pillai [Pi1]. Recent work has focused on finding small upper bounds on N . The case $rs=1$ has received particular attention. Le [Le] showed that Later, Bennett [Be] relaxed these restrictions to ... and later the authors [SS2] removed the restrictions on The results of [Be] and [SS2] also Later, Le [Le] obtained the following improved value for N :

Theorem [Le]: If $a>0$ ”

Notice that we usually explicitly give the name of the person referenced (“mathematicians are vain”) and generally put the reference right after the name. The reference is generally a couple letters or numbers in brackets, not the entire name (“mathematicians are lazy”). Math papers never¹ use footnotes, since if I² footnote an equation like $a^2 + b^2 = c^2$, the superscript looks like a power. And mathematicians refuse to footnote practically every sentence and certainly never use *Ibid*, unlike many humanities papers (“mathematicians are lazy”); although this excerpt does not show it, if we had a paragraph or section all dealing with the [Le] reference, we would only reference the beginning of the paragraph or section, and the reader would assume that Maohua Le is credited with everything in the paragraph unless the author indicates otherwise. Note also that multiple authored papers are referenced by a letter from each name (“mathematicians are vain”, e.g., above the [SS2] recognizes both authors Scott and Styer). Multiple papers by one author are listed chronologically in the References (e.g., the [Pi1] reference is the first of several papers by Pillai, of which this one has the earliest date). The major concern in mathematics papers is to correctly attribute results to the proper discoverer (“mathematicians are vain”).

Definitions / Theorems / Proofs: In formal math papers, the content of the definition or theorem is typeset in italics to make it stand out (you will notice this in the math articles you will read). But for our purposes we will be lazy and simply boldface the Definition or Theorem or Lemma or Proof headings to make it stand out.

Example:

Definition: Define the n th power sum of the digits function, $S_n(x)$, as $S_n(x) = a_r^n + \dots + a_1^n + a_0^n$ where $x = a_r 10^r + \dots + a_1 10 + a_0$ is the base ten representation of x .

The usual convention is that a Lemma is a result that is used to prove a Theorem, a Theorem is a big result, a Corollary is a result that immediately follows from the Theorem, and a Proposition is ambiguous.

¹ Never use this type of footnote.

² Doesn't this look like I squared rather than a footnoted first person pronoun?

Note that math papers are much like Powerpoint bulleted presentations: the definitions and theorems and section headings make it easy to discern the organization, and make it easy to skim-read for the major ideas without actually reading the proofs in detail.

More writing advice:

A Primer of Mathematical Writing, by Steven Krantz

Writing Mathematics Well, by Leonard Gillman

Journal of Integer Sequences LaTeX Style Guide <https://cs.uwaterloo.ca/journals/JIS/texrecs.pdf>

Finding articles on your topic:

- Our textbook of course! Next see the references at the end of the papers that Guy lists.
- MathSciNet (as our research librarian Mr. Alfred Fry will show you) (best for standard number theory topics)
- For recreational number theory topics: Wikipedia references, MathWorld references, and references in the OEIS (Online Encyclopedia of Online Sequences).

Grading Criteria:

1. Attendance:

if miss more than four classes, drop a grade
complete all peer evaluation components

2. Clarity of the math:

definitions are precise and appropriate
notation is simple, precise, and not confusing
theorems carefully stated: give all hypotheses and give precise result
proofs are correct and are logically organized into small units
conjectures are not obviously provable and are supported by some evidence

3. Presentations:

personal presence: appearance and gestures not distracting
voice clear and confident
Powerpoint, blackboard, handouts, Maple, etc. used effectively
(e.g., choice of colors and font sizes, graphics, logical layout of slides)
timing appropriate, level of detail appropriate
graphs and formulas and tables appropriate
(communicate the sense without overwhelming the listener)

4. Paper:

length fifteen to twenty pages of text, with formal definitions and theorems.
in addition, include appendices of, e.g., computer programs and output
organized into sections
introduction outlines sections and highlights main results
writing style stresses clarity and organization
references in a proper format

5. Mathematical Depth

“love covers a multitude of sins”—lovely results will impress me highly! This qualitative excellence will often determine an A- paper from a B+ paper.