

Dear Gerry,

We would like to submit a reworded version of the WCNT problem 007:03 we submitted last December, if it is convenient. This rewording clears up possible confusion, but is equivalent to the original question. We hope this is in time to include in the updated list of problems that will be sent with the next West Coast Number Theory conference mailing.

Thanks so much for all your valuable work on keeping track of the WCNT problem list. We look forward to receiving the next update of the problems.

Sincerely, Bob Styer and Reese Scott

PS:  $\neq$  is the not equal sign.

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Find all solutions to  $a^x \pm a^y = b^r \pm b^s$ , the signs being chosen independently.

Remarks: We assume  $a, b > 1$ ,  $a \neq b$ ,  $a$  and  $b$  not perfect powers,  $x > y > 0$ ,  $r > s > 0$ , and  $x/y \neq r/s$ , to eliminate infinite families. Below is a list of 23 solutions from which all other known solutions can be derived, either by combining two solutions to create a third (e.g.,  $2^5 - 2 = 3^3 + 3 = 5^2 + 5$ ,  $2^8 - 2^2 = 3^5 + 3^2 = 6^3 + 6^2$ ), or by noting that from any solution with  $x=2y$  we can derive another solution from  $a^{2y} \pm a^y = (a^y \pm 1)^2 \mp (a^y \pm 1)$ .

The 23 solutions are:

$$2^3 - 2 = 3^2 - 3$$

$$2^5 - 2^3 = 3^3 - 3$$

$$2^8 - 2^4 = 3^5 - 3$$

$$2^7 - 2^3 = 5^3 - 5$$

$$2^4 + 2^3 = 3^3 - 3$$

$$2^4 + 2 = 3^3 - 3^2$$

$$2^5 - 2 = 3^3 + 3$$

$$2^8 - 2^2 = 3^5 + 3^2$$

$$2^3 + 2^2 = 3^2 + 3$$

$$2^5 + 2^2 = 3^3 + 3^2$$

$$2^7 + 2 = 5^3 + 5$$

$$2^7 + 2^2 = 11^2 + 11$$

$$3^3 + 3 = 5^2 + 5$$

$$3^7 - 3 = 13^3 - 13$$

$$2^{13} - 2 = 91^2 - 91$$

$$5^7 - 5 = 279^2 + 279$$

$$3^5 + 3^2 = 6^3 + 6^2$$

$$3^8 - 3^4 = 6^5 - 6^4$$

$$6^3 - 6 = 15^2 - 15$$

$$5^5 + 5^2 = 15^3 - 15^2$$

$$2^{16} + 2^6 = 40^3 + 40^2$$

$$21^3 + 21^2 = 98^2 + 98$$

$$30^5 - 30 = 4929^2 + 4929$$

Are there other solutions? There are no others with terms less than  $10^{20}$  when  $a, b < 53000$ .