

6
6
6
6
7
7
-1
-1

(6)

> *evalf*(*log10*(*N*));

100.4607325

(7)

> $101 \cdot 9^3$;

73629

(8)

so we do need to check six in a two possibilities up to almost 75000.

We check for four or more in a row, beginning with $a=220000$ (where we finished with the least five in a row analysis we have already completed).

>

> **for** *a* **from** 22000 **to** 50000 **do**

for *d1* **from** 0 **to** 9 **do**

s := (1 - *a* - *d1*) **mod** 3;

for *d0* **from** *s* **to** 9 **by** 3 **do**

if *happy*(*a* + *onestep*(*d1*·10 + *d0*)) > 0 **and** *happy*(*a* + *onestep*(*d1*·10 + *d0* + 3)) > 0
 and *happy*(*a* + *onestep*(*d1*·10 + *d0* + 6)) > 0 **and** *happy*(*a* + *onestep*(*d1*·10 + *d0*
 + 9)) > 0 **then** *print*(*a*, *d1*, *d0*, *happy*(*a* + *onestep*(*d1*·10 + *d0* + 12)), *happy*(*a*
 + *onestep*(*d1*·10 + *d0* + 15))); **end if**;

end do;

end do;

end do;

31404, 6, 7, -1, -1

37404, 6, 7, -1, -1

39547, 4, 5, -1, 4

45405, 3, 4, -1, -1

45703, 3, 3, -1, -1

45722, 2, 3, -1, -1

(9)

> **for** *a* **from** 50000 **to** 75000 **do**

for *d1* **from** 0 **to** 9 **do**

s := (1 - *a* - *d1*) **mod** 3;

for *d0* **from** *s* **to** 9 **by** 3 **do**

if *happy*(*a* + *onestep*(*d1*·10 + *d0*)) > 0 **and** *happy*(*a* + *onestep*(*d1*·10 + *d0* + 3)) > 0
 and *happy*(*a* + *onestep*(*d1*·10 + *d0* + 6)) > 0 **and** *happy*(*a* + *onestep*(*d1*·10 + *d0*
 + 9)) > 0 **then** *print*(*a*, *d1*, *d0*, *happy*(*a* + *onestep*(*d1*·10 + *d0* + 12)), *happy*(*a*
 + *onestep*(*d1*·10 + *d0* + 15))); **end if**;

end do;

end do;

end do;

54405, 3, 4, -1, -1

```

54740, 2, 3, -1, -1
54748, 0, 3, -1, -1
55215, 6, 7, -1, -1
69737, 5, 6, -1, -1
69854, 5, 6, -1, -1
69854, 8, 9, -1, -1
70062, 1, 6, -1, -1
70582, 1, 2, -1, -1
70736, 8, 9, -1, -1
71464, 1, 2, -1, -1
73404, 6, 7, -1, -1
73746, 9, 7, -1, -1

```

(10)

These results show there is no five in a row without a carry. If there is a 4/2 split or a 2/4 split about the carry, it could only occur with N having last digits 88, 89, 90, or 00, 01, 02; only two of the above satisfy this, a=69854 and 70736.

```

> a := 69854;
   ktop := iquo(a, 93);
   for k from 0 to ktop do # k is the number of nines at the end of N1.
     for d2 from 0 to 8 do # d2 is the digit that precedes the k digits of nine.
       if happy(a + (d2 + 1)3 - d23 - 93·k + 13) > 0 and happy(a + (d2 + 1)3 - d23 - 93·k
         + 43) > 0 then print(k, d2, happy(a + (d2 + 1)3 - d23 - 93·k + 73)) end if;
     end do; end do:

```

```

a := 69854
ktop := 95
1, 1, -1
72, 1, -1

```

(11)

```

> a := 70736;
   ktop := iquo(a, 93);
   for k from 0 to ktop do # k is the number of nines at the end of N1.
     for d2 from 0 to 8 do # d2 is the digit that precedes the k digits of nine.
       if happy(a + (d2 + 1)3 - d23 - 93·k + 13) > 0 and happy(a + (d2 + 1)3 - d23 - 93·k
         + 43) > 0 then print(k, d2, happy(a + (d2 + 1)3 - d23 - 93·k + 73)) end if;
     end do; end do:

```

```

a := 70736
ktop := 97

```

(12)

We use our LowCubics program to investigate the a=69854 cases, and the smallest example comes from k=72, d2 = 1, giving us the 101 digit number N = 28888.(21 nines).1.(72 nines).89.

```

>
>
>

```

We will now check for a 3/3 split over the carry. Then the last digits of N must be 91 or 92 or 93.

```

> for d0 from 1 to 3 do
   s := (1 - d0) mod 3;
   for a from s + 21000 to 70000 by 3 do

```

```

if happy(a + onestep(90 + d0)) > 0 and happy(a + onestep(90 + d0 + 3)) > 0
  and happy(a + onestep(90 + d0 + 6)) > 0 then print( a, d0, happy(a + onestep(90
    + d0 - 3))); end if;
end do;
end do;

```

```

69854, 2, 5
45001, 3, -1
54019, 3, -1

```

(13)

> We check the 45001 and 54019 for 3/3 split possibilities:

```

> a := 45001;
 ktop := iquo(a, 93);
  for k from 0 to ktop do # k is the number of nines at the end of N1.
  for d2 from 0 to 8 do # d2 is the digit that precedes the k digits of nine.
  if happy(a + (d2 + 1)3 - d23 - 93·k + 23) > 0 and happy(a + (d2 + 1)3 - d23 - 93·k
    + 53) > 0 then print(k, d2, happy(a + (d2 + 1)3 - d23 - 93·k + 83)) end if;
  end do; end do;

```

```

a := 45001
ktop := 61
34, 5, -1
36, 2, -1

```

(14)

```

> a := 54019;
 ktop := iquo(a, 93);
  for k from 0 to ktop do # k is the number of nines at the end of N1.
  for d2 from 0 to 8 do # d2 is the digit that precedes the k digits of nine.
  if happy(a + (d2 + 1)3 - d23 - 93·k + 23) > 0 and happy(a + (d2 + 1)3 - d23 - 93·k
    + 53) > 0 then print(k, d2, happy(a + (d2 + 1)3 - d23 - 93·k + 83)) end if;
  end do; end do;

```

```

a := 54019
ktop := 74
36, 1, -1
70, 6, -1

```

(15)

so the 45001 and the 54019 cannot have three after the carry.

Thus, our N above is the smallest example.

```

>
>

```