Strings of Consecutive Happy Numbers
23 Feb 2008
1 Mar 2008
15 Mar 2008
21 Mar 2008
1 Sept 2008
8 Sept 2008
27 Jun 2009
25 July 2009

The goal here is to show that the first string of twelve consecutive happy numbers.

We claim the 158162 digit number N = 388.(158021 {\rm \ nines}).8.(136 {\rm \ nines}).4 begins a sequence of 12 consecutive happy numbers.

Note that Dr. Grundman uses S_2 but we will simply use S in the explanation, which in our program is the procedure 'onestep'.

Also note we will use a dot as the digit concatenation operator. Thus, 111112999994 could be written 11111.2.9999.94 if we wish.

First we define our procedures.

```
> restart;
> f := n → n^2;  #in case we ever want to investigate the cube of the digits, etc.
```

$$f := n \to n^2 \quad \textbf{(1)}$$

```
> bs := 10;
      #this is the base, in case we ever want to investigate binary or ternary or any other base.
```

$$bs := 10 \quad \textbf{(2)}$$

```
> onestep := proc(n1)
      #this is what Dr. Grundman calls S_2(n1) and what we will simply call S below.
    local ans, n, d;
    n := n1;
    ans := 0;
    while n > 0 do
    d := n mod bs;
    ans := ans + f(d);
    n := (n-d)/bs;
    end do;
    ans;
    end;
```

```
onestep := proc(n1)                                                    (3)
    local ans, n, d;
    n := n1;
    ans := 0;
    while 0 < n do d := mod(n, bs); ans := ans + f(d); n := (n - d)/bs end do;
```

```
    ans
end proc
```

```
> happy := proc(n)
      # returns -1 if not happy, and returns the number of steps to reach 1 if it is happy
    local m, j, height;
    m := n;
    height := -1;
    for j from 1 to 100 while (m > 1 and m ≠ 4) do
    m := onestep(m);
     end do;
    if m = 1 then height := j; end if;
    height;
    end;
```

```
happy := proc(n)                                                    (4)
    local m, j, height;
    m := n;
    height := -1;
    for j to 100 while 1 < m and m <> 4 do m := onestep(m) end do;
    if m = 1 then height := j end if;
    height
end proc
```

The next procedure is only needed when we want to find the smallest N with S(N) = n for a given n. A separate worksheet has the details on how this is constructed. The array contains the smallest N for 1 <= n <= 486 = 6*81.

```
> lowS := [1, 11, 111, 2, 12, 112, 1112, 22, 3, 13, 113, 222, 23, 123, 1123, 4, 14, 33, 133, 24,
      124, 233, 1233, 224, 5, 15, 115, 1115, 25, 125, 1125, 44, 144, 35, 135, 6, 16, 116, 1116,
      26, 45, 145, 335, 226, 36, 136, 1136, 444, 7, 17, 117, 46, 27, 127, 1127, 246, 227, 37, 137,
      1137, 56, 156, 1156, 8, 18, 118, 337, 28, 128, 356, 1356, 66, 38, 57, 157, 266, 238, 257,
      1257, 48, 9, 19, 119, 248, 29, 129, 1129, 466, 58, 39, 139, 1139, 258, 239, 1239, 448, 49,
      77, 177, 68, 168, 277, 1277, 268, 458, 59, 159, 666, 368, 259, 1259, 2666, 78, 178, 359,
      468, 69, 169, 1169, 2468, 269, 378, 577, 1577, 568, 369, 1369, 88, 188, 79, 179, 288, 469,
      279, 1279, 668, 388, 578, 379, 1379, 2388, 569, 1569, 488, 89, 189, 777, 1777, 289, 1289,
      2777, 4668, 588, 389, 579, 1579, 2588, 2389, 2579, 4488, 489, 99, 199, 688, 1688, 299,
      1299, 2688, 4588, 589, 399, 1399, 3688, 2589, 2399, 12399, 788, 499, 779, 1779, 689,
      1689, 2779, 12779, 2689, 3788, 599, 1599, 5688, 3689, 2599, 888, 1888, 789, 1789, 2888,
      4689, 699, 1699, 6688, 3888, 2699, 3789, 5779, 15779, 5689, 3699, 4888, 889, 1889, 799,
      1799, 2889, 4699, 2799, 12799, 5888, 3889, 5789, 3799, 13799, 23889, 5699, 15699,
      4889, 899, 1899, 6888, 16888, 2899, 12899, 26888, 45888, 5889, 3899, 5799, 15799,
      25889, 23899, 25799, 7888, 4899, 999, 1999, 6889, 16889, 2999, 12999, 26889, 37888,
      5899, 3999, 13999, 36889, 25899, 8888, 18888, 7889, 4999, 7799, 17799, 6899, 16899,
      27799, 38888, 26899, 37889, 5999, 15999, 56889, 36899, 25999, 8889, 18889, 7899,
      17899, 28889, 46899, 6999, 16999, 58888, 38889, 26999, 37899, 57799, 157799, 56899,
      36999, 48889, 8899, 18899, 7999, 17999, 28899, 46999, 27999, 127999, 58889, 38899,
      57899, 37999, 137999, 238899, 56999, 78888, 48899, 8999, 18999, 68889, 168889,
      28999, 128999, 268889, 378888, 58899, 38999, 57999, 157999, 258899, 88888, 188888,
      78889, 48999, 9999, 19999, 68899, 168899, 29999, 129999, 268899, 378889, 58999,
      39999, 139999, 368899, 258999, 88889, 188889, 78899, 49999, 77999, 177999, 68999,
```

```
        168999, 277999, 388889, 268999, 378899, 59999, 159999, 568899, 368999, 259999,
        88899, 188899, 78999, 178999, 288899, 468999, 69999, 169999, 588889, 388899,
        269999, 378999, 577999, 1577999, 568999, 369999, 488899, 88999, 188999, 79999,
        179999, 288999, 469999, 279999, 1279999, 588899, 388999, 578999, 379999, 1379999,
        888888, 569999, 788889, 488999, 89999, 189999, 688899, 1688899, 289999, 1289999,
        2688899, 3788889, 588999, 389999, 579999, 1579999, 2588999, 888889, 1888889,
        788899, 489999, 99999, 199999, 688999, 1688999, 299999, 1299999, 2688999, 3788899,
        589999, 399999, 1399999, 3688999, 2589999, 888899, 1888899, 788999, 499999,
        779999, 1779999, 689999, 1689999, 2779999, 3888899, 2689999, 3788999, 599999,
        1599999, 5688999, 3689999, 2599999, 888999, 1888999, 789999, 1789999, 2888999,
        4689999, 699999, 1699999, 5888899, 3888999, 2699999, 3789999, 5779999, 8888888,
        5689999, 3699999, 4888999, 889999, 1889999, 799999, 1799999, 2889999, 4699999,
        2799999, 12799999, 5888999, 3889999, 5789999, 3799999, 13799999, 8888889,
        5699999, 7888899, 4889999, 899999, 1899999, 6888999, 16888999, 2899999, 12899999,
        26888999, 37888899, 5889999, 3899999, 5799999, 15799999, 25889999, 8888899,
        18888899, 7888999, 4899999, 999999]:
> minimalN := proc(n)
   local q, r, k, ans;
  global lowS;;
    if n < 487 then ans := lowS[n];
   else
  q := iquo(n, 81,'r');
   ans := lowS[n - (q - 5)·81]·10^(q - 5) + (10^(q - 5) - 1);
   end if;
   ans;
   end;
```

$$\mathit{minimalN} := \mathbf{proc}(n) \quad \textbf{(5)}$$

```
    local q, r, k, ans;
    global lowS;
    if n < 487 then
        ans := lowS[n]
    else
        q := iquo(n, 81, 'r'); ans := lowS[n - 81 * q + 405] * 10^(q - 5) + 10^(q - 5) - 1
    end if;
    ans
end proc
```

We claim the 158162 digit number N = 388.(158021 {\rm \ nines}).8.(136 {\rm \ nines}).4 begins a sequence of 12 consecutive happy numbers.

We first show this does give twelve in a row.

```
> N := 388·10^158159 + (10^158021 - 1)·10^138 + 8·10^137 + (10^136 - 1)·10 + 4
```

$$N := \quad \textbf{(6)}$$

```
3889999999999999999999999999999999999999999999999999999999999999999999999999\
9999999999999999999999999[...157962 digits...]
9999999999999999999999999999999999999999999999999999999999999999999999999999
99999999999999999999999994
```

First we check for six in a row before the carry, then six in a row after the carry.

```
> M1 := onestep(388) + 158021·9^2 + 8^2 + 136·9^2;
  for j from 3 to 9 do print( j, happy(M1 + j^2) ); end do;
```

$$M1 := 12810918$$

$$3, -1$$
$$4, 5$$
$$5, 5$$
$$6, 5$$
$$7, 7$$
$$8, 5$$
$$9, 7 \qquad \textbf{(7)}$$

```
> M1 := onestep(388) + 158021·9^2 + (8 + 1)^2;
  for j from 0 to 6 do print( j, happy(M1 + j^2) ); end do;
```

$$M1 := 12799919$$

$$0, 8$$
$$1, 4$$
$$2, 4$$
$$3, 8$$
$$4, 7$$
$$5, 7$$
$$6, -1 \qquad \textbf{(8)}$$

We first search for six in a row before the carry.

```
> for a from 1 to 13000000 do
    if happy(a + 9^2) > 0 and happy(a + 8^2) > 0 and happy(a + 7^2) > 0 and happy(a + 6^2)
       > 0 and happy(a + 5^2) > 0 and happy(a + 4^2) > 0 then print(a, happy(a + 3^2),
       happy(a + 2^2)); end if;
  end do:
```

$$126980, -1, -1$$
$$216980, -1, -1$$
$$226056, -1, -1$$
$$262056, -1, -1$$
$$277718, -1, -1$$
$$356918, -1, -1$$
$$359618, -1, -1$$
$$365918, -1, -1$$
$$369518, -1, -1$$
$$395618, -1, -1$$
$$396518, -1, -1$$
$$451956, -1, -1$$
$$536918, -1, -1$$
$$539618, -1, -1$$
$$541956, -1, -1$$
$$563918, -1, -1$$

569318, −1, −1
593618, −1, −1
596318, −1, −1
622056, −1, −1
635918, −1, −1
639518, −1, −1
653918, −1, −1
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695318, −1, −1
727718, −1, −1
772718, −1, −1
777218, −1, −1
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12970418, -1, -1
12974018, -1, -1
12980118, -1, -1
12981018, -1, -1 **(9)**

```
> for a from 1 to 13000000 do
  if happy(a) > 0 and happy(a + 1^2) > 0 and happy(a + 2^2) > 0 and happy(a + 3^2) > 0
     and happy(a + 4^2) > 0 and happy(a + 5^2) > 0 then print(a, happy(a + 6^2), happy(a
     + 7^2)); end if;
  end do:
```

12799919, -1, 8
12979919, -1, 8
12997919, -1, 8
12999719, -1, 8 **(10)**

```
> a;
```

13000001 **(11)**

We see that there are not seven in a row before the carry, nor seven in a row after the carry, so the only way to get 12 in a row is to have a 6/6 split over the carry.

We test the smallest case which has six after the carry to see if it can extend to six in a row before the carry.

First we see which digit will give us the smallest candidate. We see here our preference is d=8 though the difference in length would not be much.

```
> trunc(12799919/81)
```

158023 **(12)**

```
> a := 12799919 :
  for d from 0 to 8 do
  m := a - 158018·9^2 - d^2;
  print(d, minimalN(m));
  end do:
```

0, 3889999
1, 5888999
2, 4699999
3, 889999
4, 2699999
5, 1888999
6, 1689999
7, 3788899
8, 389999 **(13)**

We now add digits onto the minimal case of six in a row after the carry to see if we can get 6 in a row before the carry.

```
> a := 12799919;
  ktop := 1000;
   for k from 0 to ktop do  # k is the number of nines at the end of N1.
  for d from 0 to 8 do   # d is the digit that precedes the k digits of nine at the end of N1.
```

```
q := a-2·d − 1 + k·9^2;
if happy(q + 9^2) > 0 and happy(q + 8^2) > 0 and happy(q + 7^2) > 0 and happy(q + 6^2)
    > 0 and happy(q + 5^2) > 0 then print(a, k, d, happy(q + 4^2)) end if;
end do; end do:
```

$$a := 12799919$$

$$ktop := 1000$$

12799919, 136, 8, 5

12799919, 236, 8, 5 **(14)**

The smaller N comes from the smaller k = 136. Since 12799919+136*81 < 12979919 . we can ignore the higher cases. So our candidate is N= 38.(158021 nines).8.(136 nines).4. As we saw above, this works, there can be no smaller with six in a row after the carry, so it is the smallest.

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