Practice Third Midterm Math 1335

5 April 2006

Please show all your work and relevant ideas.

1. Consider the Cobb-Douglas production function $P(L, C) = 3.5L^{0.4}C^{0.6}$ where P is the production in millions of dollars, L is the labor expenditures in millions of dollars, and C is the annual capital investment in millions of dollars.

Find the marginal productivity with respect to labor. Calculate this when L=10 and C=30, then again with L=20 and C=20.

Find the marginal productivity with respect to capital. Calculate this when L=10 and C=30, then again with L=20 and C=20.

You have a million dollars to invest in either labor or capital. For L=10 and C=30, then again with L=20 and C=20, which should you invest in and why?

2. Find these partial derivatives for this volume function V:

$$V(x, y, z, \lambda) = xyz - 2\lambda x - 2\lambda y - \lambda z + 100\lambda$$

(a.) $\frac{\partial V}{\partial x}$

(b.) $V_z(1, 2, 3, 4)$

(c.) V_λ

(d.) $\frac{\partial}{\partial x}\frac{\partial V}{\partial \lambda}(1, 2, 3, 4)$

(e.) V_{yz}

3. Find these partial derivatives for this profit function π:

$$\pi(s, p) = 27s - 10s^2 - 3sp + 35p - 25p^2 + 140$$

(a.) $\frac{\partial \pi}{\partial s}$

(b.) $\pi_p(2, 3)$

(c.) π_{ss}

(d.) $\frac{\partial}{\partial p}\frac{\partial \pi}{\partial s}(2, 3)$

(e.) π_{ps}

Section 8.2 numbers 2, 4, 6, 12, 14, 16, 18

4. Looking at the coefficients of these quadratic functions, you should be able to tell if this is mountain-shaped (maximum), cup-shaped (minimum), or saddle-shaped. Briefly indicate which and why.

(a.) $-3.262x^2 + 2.48x - 13.5y^2 + 2.69xy + 14.0y + 152$

(b.) $3.262x^2 - 2.48x + 2.69xy + 14.0y - 13.5y^2 - 152$

(c.) $3.262x^2 - 2.48x + 13.5y^2 - 2.69xy - 14.0y + 152$

(d.) $0.262x^2 + 21.48x + 0.135y^2 + 26.98xy + 1.40y + 152$

(e.) $-0.262x^2 + 21.48x - 0.135y^2 + 26.98xy + 1.40y + 152$

(d.) $0.262x^2 + 21.48x - 0.135y^2 + 26.98xy + 1.40y + 152$

(e.) $-0.0326x^2 + 24.48x - 0.0135y^2 + 0.0069xy + 14.0y + 12$

(f.) $0.0326x^2 - 24.48x - 0.0135y^2 + 0.0069xy - 14.0y + 12$

(g.) $-0.0326x^2 - 24.48x + 0.0135y^2 - 0.0069xy + 14.0y + 12$

(h.) $0.0326x^2 + 24.48x + 0.0135y^2 + 0.0069xy + 14.0y + 12$

5. Consider these matrices:

$$A = \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \\ x_4 & y_4 & 1 \\ x_5 & y_5 & 1 \\ x_6 & y_6 & 1 \\ x_7 & y_7 & 1 \end{bmatrix}, M = \begin{bmatrix} a \\ b \\ c \end{bmatrix}, Z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \\ z_6 \\ z_7 \end{bmatrix}$$

Find $A^T \cdot A$ and $A^T \cdot Z$.

6. Refer to the previous problem. Show the algebra that gets us from $AM \sim Z$ to the formula that is used to calculate the regression coefficients, namely, $M = (A^T \cdot A)^{-1} A^T \cdot Z$. Why is it wrong to solve for M via $M = A^{-1}Z$?