Practice Third Midterm Math 1335

5 April 2006

Please show all your work and relevant ideas.

1. Consider the Cobb-Douglas production function \( P(L, C) = 3.5L^{0.4}C^{0.6} \) where \( P \) is the production in millions of dollars, \( L \) is the labor expenditures in millions of dollars, and \( C \) is the annual capital investment in millions of dollars.

Find the marginal productivity with respect to labor. Calculate this when \( L=10 \) and \( C=30 \), then again with \( L=20 \) and \( C=20 \).

Find the marginal productivity with respect to capital. Calculate this when \( L=10 \) and \( C=30 \), then again with \( L=20 \) and \( C=20 \).

You have a million dollars to invest in either labor or capital. For \( L=10 \) and \( C=30 \), then again with \( L=20 \) and \( C=20 \), which should you invest in and why?

2. Find these partial derivatives for this volume function \( V \):

\[
V(x, y, z, \lambda) = xyz - 2\lambda x - 2\lambda y - \lambda z + 100\lambda
\]

(a.) \( \frac{\partial V}{\partial x} \)

(b.) \( V_z(x, y, z, \lambda) \bigg|_{(1,2,3,4)} \)

(c.) \( V_\lambda \)

(d.) \( \frac{\partial}{\partial x} \frac{\partial V}{\partial \lambda} \bigg|_{(1,2,3,4)} \)

(e.) \( V_{yz} \)

3. Find these partial derivatives for this profit function \( \pi \):

\[
\pi(s, p) = 27s - 10s^2 - 3sp + 35p - 25p^2 + 140
\]

(a.) \( \frac{\partial \pi}{\partial s} \)

(b.) \( \pi_p(2, 3) \)

(c.) \( \pi_{ss} \)

(d.) \( \frac{\partial}{\partial p} \frac{\partial \pi}{\partial s} \bigg|_{(2,3)} \)

(e.) \( \pi_{ps} \)

Section 8.2 numbers 2, 4, 6, 12, 14, 16, 18
4. Looking at the coefficients of these quadratic functions, you should be able to tell if this is mountain-shaped (maximum), cup-shaped (minimum), or saddle-shaped. Briefly indicate which and why.

(a.) \(-3.262x^2 + 2.48x - 13.5y^2 + 2.69xy + 14.0y + 152\)

(b.) \(3.262x^2 - 2.48x + 2.69xy + 14.0y - 13.5y^2 - 152\)

(c.) \(3.262x^2 - 2.48x + 13.5y^2 - 2.69xy - 14.0y + 152\)

(d.) \(0.262x^2 + 21.48x + 0.135y^2 + 26.98xy + 1.40y + 152\)

(e.) \(-0.262x^2 + 21.48x - 0.135y^2 + 26.98xy + 1.40y + 152\)

(f.) \(0.262x^2 + 21.48x - 0.135y^2 + 26.98xy + 1.40y + 152\)

(g.) \(-0.0326x^2 + 24.48x - 0.0135y^2 + 0.0069xy + 14.0y + 12\)

(h.) \(0.0326x^2 - 24.48x - 0.0135y^2 + 0.0069xy + 14.0y + 12\)

5. Consider these matrices:

\[
A = \begin{bmatrix}
x_1 & y_1 & 1 \\
x_2 & y_2 & 1 \\
x_3 & y_3 & 1 \\
x_4 & y_4 & 1 \\
x_5 & y_5 & 1 \\
x_6 & y_6 & 1 \\
x_7 & y_7 & 1 \\
\end{bmatrix}, \quad M = \begin{bmatrix} a \\ b \\ c \end{bmatrix}, \quad Z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \\ z_6 \\ z_7 \end{bmatrix}
\]

Find \(A^T \cdot A\) and \(A^T \cdot Z\).

6. Refer to the previous problem. Show the algebra that gets us from \(AM \sim Z\) to the formula that is used to calculate the regression coefficients, namely, \(M = (A^T \cdot A)^{-1} A^T \cdot Z\). Why is it wrong to solve for \(M\) via \(M = A^{-1} Z\)?