Third Midterm Exam Math 1335, 10 April 2006, Section 11

Please show all your work and relevant ideas.

1. Consider the Cobb-Douglas production function $P(L, C) = 5.1L^{0.333}C^{0.667}$ where P is the production in millions of dollars, L is the labor expenditures in millions of dollars, and C is the annual capital investment in millions of dollars.

(a.) Find the marginal productivity with respect to labor and calculate this when $L = 20$ and $C = 30$.

(b.) Find the marginal productivity with respect to capital and calculate this when $L = 20$ and $C = 30$.

You have a million dollars to invest in either labor or capital. Which should you invest in and why?

2. Find these partial derivatives for this profit function π:

$$\pi(s, t) = 17s - 12s^2 - 2st + 40t - 21t^2 + 140$$

(a.) $\frac{\partial \pi}{\partial t}(3, 2)$

(b.) π_s

(c.) π_{ss}

(d.) $\frac{\partial}{\partial s}\frac{\partial \pi}{\partial t}(3, 2)$

3. Find the critical points for this function:

$$f(w, y) = 20w^2 + 30y^2 - 50wy + 20w - 40y + 12321$$

4. Looking at the coefficients of these quadratic functions, you should be able to tell if this is mountain-shaped (maximum), cup-shaped (minimum), or saddle-shaped. Briefly indicate which and why.

(a.) $-32.8a^2 - 23.5b^2 + 2.69ab + 31.5a + 14.2b - 152$

(b.) $32.8a^2 - 23.5b^2 - 2.69ab - 31.5a - 14.2b - 152$

(c.) $32.8a^2 + 23.5b^2 + 2.69ab + 31.5a - 14.2b + 152$

(d.) $0.0262p^2 + 2.48p - 0.0135q^2 + 2.90pq + 1.84q + 1052$

(e.) $0.0262p^2 - 2.48p + 0.0135q^2 + 2.90pq - 1.84q + 1052$

(f.) $-0.0262p^2 - 2.48p - 0.0135q^2 - 2.90pq - 1.84q + 1052$

5. Consider these matrices:

$$C = \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \\ x_4 & y_4 & 1 \\ x_5 & y_5 & 1 \\ x_6 & y_6 & 1 \\ x_7 & y_7 & 1 \\ x_8 & y_8 & 1 \end{bmatrix}, U = \begin{bmatrix} a \\ b \\ c \end{bmatrix}, P = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \\ z_6 \\ z_7 \\ z_8 \end{bmatrix}$$

Find $C^T \cdot C$, $C \cdot U$, and $C^T \cdot P$.

6. Refer to the previous problem. Show the algebra that gets us from $CU \sim P$ to the formula that is used to calculate the regression coefficients, namely, $U = (C^T \cdot C)^{-1} C^T \cdot P$. Why is it wrong to solve for U via $U = C^{-1}P$?