

Third Midterm Exam Math 1335, 10 April 2006, Version 1

Please show all your work and relevant ideas.

1. Consider the Cobb-Douglas production function $P(L, C) = 4.6L^{0.3}C^{0.7}$ where P is the production in millions of dollars, L is the labor expenditures in millions of dollars, and C is the annual capital investment in millions of dollars.

(a.) Find the marginal productivity with respect to labor and calculate this when $L = 20$ and $C = 50$.

(b.) Find the marginal productivity with respect to capital and calculate this when $L = 20$ and $C = 50$.

You have a million dollars to invest in either labor or capital. Which should you invest in and why?

2. Find these partial derivatives for this volume function V :

$$V(x, y, z, \lambda) = x^2y^2z^2 - 3\lambda x^2 - 3\lambda y^2 - \lambda z^2 + 100\lambda^2$$

(a.) $\frac{\partial V}{\partial \lambda}$

(b.) $V_x(4, 2, 3, 1)$

(c.) $\frac{\partial}{\partial y} \frac{\partial V}{\partial x}(4, 2, 3, 1)$

(d.) $V_{x\lambda}$

3. Find the critical points for this function:

$$f(m, n) = 60m^2 + 10n^2 - 50mn + 40m - 30n + 1221$$

4. Looking at the coefficients of these quadratic functions, you should be able to tell if this is mountain-shaped (maximum), cup-shaped (minimum), or saddle-shaped. Briefly indicate which and why.

(a.) $-0.0361x^2 + 2.899x - 0.0157y^2 + 0.68xy + 1.401y + 12.45$

(b.) $0.0361x^2 + 2.899x + 0.0157y^2 - 0.68xy + 1.401y + 12.45$

(c.) $0.0361x^2 - 2.899x - 0.0157y^2 + 0.68xy - 1.401y + 12.45$

(d.) $-0.361x^2 + 244.8x - 0.157y^2 + 0.0068xy + 140.1y + 1245$

(e.) $0.361x^2 - 244.8x + 0.157y^2 + 0.0068xy - 140.1y + 1245$

(f.) $-0.361x^2 + 244.8x + 0.157y^2 - 0.0068xy + 140.1y - 1245$

5. Consider these matrices:

$$A = \begin{bmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \\ a_4 & b_4 & 1 \\ a_5 & b_5 & 1 \\ a_6 & b_6 & 1 \\ a_7 & b_7 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, C = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \end{bmatrix}$$

Find $A^T \cdot A$ and $A^T \cdot C$.

6. Refer to the previous problem. Show the algebra that gets us from $AX \sim C$ to the formula that is used to calculate the regression coefficients, namely, $X = (A^T \cdot A)^{-1} A^T \cdot C$. Why is it wrong to solve for X via $X = A^{-1}C$?