Practice Fourth Exam Math 1330 , 27 Nov 2006

Please show all your work. And please *relax*, **RELAX**, relax...

1. Halogen bulbs often have a listed (mean) lifetime about 2000 hours. The probability distribution for the actual lifetime of one of these bulbs is

$$f(t) = \frac{1}{2000} e^{-t/2000}$$

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(a.) What does the integral $\int_{t=0}^{t=1000} \frac{1}{2000} e^{-t/2000}\, dt$ tell us about our light bulb?

(b.) Solve this integral.

2. Arrival times often have a beta distribution shape. A typical beta distribution is $B(t) = 20t^3(1-t)$ with domain $0 \le t \le 1$.

(a.) What is the probability that the arrival will be in the first half hour ($0 \le t \le 0.5$)?

(b.) What is the mode of this distribution?

3. An example of a hypergeometric distribution is $0.01te^{-0.1t}$ with a domain $t \ge 0$. Find the mode of this distribution on this domain.

4. A distribution $f(t)$ with domain $0 \le t \le 24$ (t measured in hours since midnight) was entered by me into a calculator and I recorded these answers:

$$\int_{t=8}^{t=12} f(t)\, dt = 0.345, \int_{t=15}^{t=18} f(t)\, dt = 0.212, \int_{t=15}^{t=22} f(t)\, dt = 1.432,$$

$$\int_{t=2}^{t=8} f(t)\, dt = 0.009, \int_{t=22}^{t=24} f(t)\, dt = -0.009$$

(a.) My bad typing skills caused exactly two errors out of these five calculations. Which two are wrong and why?

(b.) Suppose that $f(t)$ determines the probability that I will take my daily vitamin at time t. Interpret in words what the three correct integrals tell us?

5. The standard normal probability distribution is also known as the bell curve. Suppose SAT scores are normally distributed. To calculate the probability that an SAT score is within one standard deviation, we can calculate the integral

$$\int_{z=-1}^{z=1} \frac{1}{\sqrt{2\pi}} e^{-z^2/2}\, dz$$

But there is no elementary antiderivative for this integrand, so one must use numerical methods.

(a.) Estimate this integral by using the Trapezoidal Rule with $n = 4$ boxes.

(b.) Compare your Trapezoidal estimate with the exact answer (which we can calculate on our calculator to be 0.6826894920). What is the absolute error and what is the relative error?

6. Same as Problem 6 but use the Midpoint Rule with $n = 4$ boxes.

7. A common type of integral (because the square root usually arises from distance via the Pythagorean theorem) is an integral like

$$\int_{x=1}^{x=3} \sqrt{x^2 + 16}\, dx$$

One can do this by hand using complicated trigonometry, but it is easy to calculate numerically.

(a.) Estimate this integral by using the Trapezoidal Rule with $n = 4$ boxes.

(b.) Compare your Trapezoidal estimate with the exact answer (which we can calculate on our calculator to be 9.00389294). What is the absolute error and what is the relative error?

8. Same as Problem 8 but use the Midpoint Rule with $n = 4$ boxes.

9. Calculate these integrals: Section 4.4, 1-10; Section 4.5, 1-10; plus these

(a.) $\int_{t=0}^{t=15} 1000e^{0.06t}\, dt$ (an irrelevant but interesting comment: this integral represents the total money you would receive from a steady stream of \$1000 per year income compounded continuously at 6% interest for 15 years.)

(b.) $\int_{t=0}^{t=15} 7000e^{-0.05t}\, dt$ (an irrelevant but interesting comment: this integral represents the present value of a steady stream of income of \$7000 per year for 15 years with market interest rates of 5%.)