Answers Practice Test • Nov 1 2006 Math 1330

(4) (b) $y = x^a \cdot (1-x)^b$ <u>Product Rule</u>

$$\frac{dy}{dx} = a \cdot x^{a-1}(1-x)^b + x^a \cdot b\,(1-x)^{b-1}(-1) = x^{a-1}(1-x)^{b-1}\left[a(1-x) - bx\right]$$

so <u>critical points</u> $\frac{dy}{dx} = 0$ when $x=0$, $x=1$, and $x = \frac{a}{a+b}$.

Note $y(0) = y(1) = 0$ whereas $y\left(\frac{a}{a+b}\right) = \left(\frac{a}{a+b}\right)^a \left(\frac{b}{a+b}\right)^b = \frac{a^a b^b}{(a+b)^{a+b}}$

is positive so it must be a max.

<u>2^ND derivative Test</u> $\frac{d^2y}{dx^2} = a\cdot(a-1)\,x^{a-2}(1-x)^b +$

$a \cdot x^{a-1}\,(b)\,(1-x)^{b-1}\,(-1) + a x^{a-1} \cdot b\,(1-x)^{b-1}(-1) + x^a \cdot b(b-1)(1-x)^{b-2}(+1)$

$$= x^{a-2} \cdot (1-x)^{b-2}\left[a(a-1)(1-x)^2 - 2ab\,x(1-x) + b(b-1)x^2\right]$$

Plug in our critical point $x=0$ or $x=1$ and $\frac{d^2y}{dx^2} = 0$

so ~~these~~ are inflection points!

But plug in $x = \frac{a}{a+b}$ critical point, so $1-x = \frac{b}{a+b}$,

then $$\frac{d^2y}{dx^2} = \left(\frac{a}{a+b}\right)^{a-2}\left(\frac{b}{a+b}\right)^{b-2}\left[a(a-1)\left(\frac{b}{a+b}\right)^2 - 2ab\left(\frac{a}{a+b}\right)\left(\frac{b}{a+b}\right) + b(b-1)\left(\frac{a}{a+b}\right)^2\right]$$

$$= \left(\frac{a}{a+b}\right)^{a-2}\left(\frac{b}{a+b}\right)^{b-2}\left[-\frac{ab^2}{(a+b)^2} - \frac{a^2b}{(a+b)^2}\right] < 0$$

$$= \frac{a^{a-1} \cdot b^{b-1}}{(a+b)^{a+b-3}}$$

so y is concave <u>down</u> at this critical point

so $\left\{x = \frac{a}{a+b},\ y = \frac{a^a b^b}{(a+b)^{a+b}}\right\}$ is a <u>maximum</u>.

4 (a) Just plug in $a=2$, $b=3$ to get max at $\left(\frac{2}{5}, 0.03456\right)$.