Practice Final Exam Math 1330 Fall 2005

Please show all your work. And please *relax*, **RELAX**, relax...

1. In class we discussed light bulbs which generally have an exponential probability distribution, ce^{-t/t_0} for some constant c.

(a.) Set up the integral and equation one would use to find the constant c.

(b.) Solve this integral and determine c.

2. Halogen bulbs often have a listed (mean) lifetime about 2000 hours. The probability distribution for these bulbs is

$$f(t) = \frac{1}{2000}e^{-t/2000}$$

and next year you will see that the mean is calculated by using the integral

$$\int_{t=0}^{t=\infty} tf(t)\,dt$$

Use integration by parts with $u = t$ to calculate this integral.

3. In the previous problem, you calculated the mean. Now calculate the median. Explain why these are not the same, and why the median is smaller than the mean.

4. A distribution that arises in analysis of binomial trials (flipping coins, a stock increasing or decreasing, a company making a profit or not) is the beta distribution, $B(x) = c * x^a(1-x)^b$ where a and b depend on the particular problem, and where the domain is $0 \le x \le 1$.

(a.) Assume $a = 2$ and $b = 1$. Calculate the value of c that makes this a valid probability, i.e., find $\int_0^1 cx^2(1-x)\,dx = 1$. HINT: the answer is $c = 12$.

(b.) Find the mode of this distribution.

5. For the beta distribution $B(x)$ in the last problem:

(a.) Find the median of this distribution.

(b.) The mean is given by

$$\int_{x=0}^{x=1} xB(x)\,dx = \int_{x=0}^{x=1} 12x^3(1-x)\,dx$$

Calculate this mean.

6. The standard normal probability distribution is also known as the bell curve. Suppose SAT scores are normally distributed. To calculate the probability that an SAT score is within one standard deviation, we can calculate the integral

$$\int_{z=-1}^{z=1} \frac{1}{\sqrt{2\pi}}e^{-z^2/2}\,dz$$

But there is no elementary antiderivative for this integrand, so one must use numerical methods.

(a.) Estimate this integral by using the Trapezoidal Rule with $n = 4$ boxes.

(b.) Compare your Trapezoidal estimate with the exact answer calculated on your calculator. What is the absolute error and what is the relative error?

7. Same as Problem 6 but use the Midpoint Rule with $n = 4$ boxes.

8. A common type of integral (because the square root usually arises from distance via the Pythagorean theorem) is an integral like

$$\int_{x=0}^{x=3} \sqrt{x^2 + 16}\, dx$$

One can do this by hand using complicated trigonometry, but it is easy to calculate numerically.

(a.) Estimate this integral by using the Trapezoidal Rule with $n = 3$ boxes.

(b.) Compare your Trapezoidal estimate with the exact answer calculated on your calculator. What is the absolute error and what is the relative error?

9. Same as Problem 8 but use the Midpoint Rule with $n = 3$ boxes.

10-13. Calculate these integrals: Section 4.4, 1-10; Section 4.5, 1-10; Section 4.8, 1-22.

14-18. Calculate these derivatives: Section 2.4, 19-38; Section 2.5, 3-30; Section 2.6, 1-20

19. Use the limit definition of derivative and show all algebra steps to find the derivative of:

$$\frac{3}{x}, \frac{1}{x^2}, 5x^2, 5x - 3, \frac{x^3}{3}$$