The Chemistry of Calculus: The Basic Formulas and Rules for Differentiation

Structure of Functions

Whenever you are faced with a function, determine its structure! Is it a *sum*, a *product*, a *quotient*, or a *function inside a function*? What parts are constants, either *additive constants* or *multiplicative constants*? Often it is helpful to put a function into a more 'power-full' form: $1/x^2 = x^{-2}$, $\sqrt{1-x^2} = (1-x^2)^{1/2}$.

'ATOMS': Basic Formulas for Differentiation

Additive Constants: $\frac{d}{dx}(c) = 0$

Powers: $\frac{d}{dx}(x^n) = nx^{n-1}$ Note: powers have variable in bottom, constant in top.

Exponentials: $\frac{d}{dx}(e^x) = e^x$ Self-replicating! Note: exponentials have variable in exponent and constant in bottom.

Logarithms: $\frac{d}{dx}\ln(x) = \frac{1}{x} = x^{-1}$

Trig functions: $\frac{d}{dx}\sin(x) = \cos(x)$, $\frac{d}{dx}\cos(x) = -\sin(x)$

'Molecular Bonds': Basic Rules for Differentiation

Multiplicative Constant Rule: $\frac{d}{dx}(c \cdot f(x)) = c\frac{d}{dx}f(x)$ (Constants are your friends!)

Sum Rule: $(f+g)' = f' + g'$ The derivative of a sum is the sum of the derivatives.

Product Rule: $(f \cdot g)' = f' \cdot g + f \cdot g'$

Quotient Rule:

$$\left(\frac{f}{g}\right) = \frac{f'g - fg'}{g^2}$$

Chain Rule: 'a function inside a function'

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

The derivative of the outside function times the derivative of the inside.

Here u is the inside function: the g**U**ts of the function, the **U**nified inside.

Special Cases that Students often Memorize:

$$\frac{d(x^2)}{dx} = 2x, \frac{d(e^{ax})}{dx} = ae^{ax}, \frac{d(b^x)}{dx} = \ln(b)b^x$$

The Chemistry of Calculus: The Basic Formulas and Rules for Integration

Structure of Functions

When integrating, you should determine the structure of the function being integrated. Is it a *sum* or is there a *function inside a function*? What parts are constants, either *additive constants* or *multiplicative constants*? If it is a product or quotient, sometimes algebra can transform the function. Often it is helpful to put a function into a more 'power-full' form: $1/(2x+3)^2 = (2x+3)^{-2}$, $\sqrt{1-x^2} = (1-x^2)^{1/2}$.

'ATOMS': Basic Formulas for Integration

Antiderivative: $\int dx = x + C$

Powers: $\int x^n \; dx = \frac{x^{n+1}}{n+1} + C \qquad$ if $n \neq -1$.

When $n = -1$: $\int \frac{1}{x} \; dx = \ln(x) + C$

Exponentials: $\int e^x \; dx = e^x + C \qquad$ Self-replicating!

Trig functions: $\int \sin(x) \; dx = -\cos(x) + C$, $\int \cos(x) \; dx = \sin(x) + C$

'Molecular Bonds': Basic Rules for Integration

Multiplicative Constant Rule: $\int c \cdot f(x) \; dx = c \int f(x) \; dx \qquad$ (Constants are your friends!)

Sum Rule: $\int (f+g) = \int f + \int g \qquad$ The integral of a sum is the sum of the integrals.

U-Substitution—the chain rule backwards: Choose 'u' to be the "united" inside function (the g**U**ts inside a function). Calculate 'du' and substitute everywhere, including the limits.

Integration by Parts—the product rule backwards:

$$\int_a^b u \; dv = uv\Big|_a^b - \int_a^b v \; du$$

Special Algebra Techniques to Transform Integrand: partial fraction decomposition, trig substitution, complex exponentials, etc.

Special Cases that Students often Memorize:

$$\int x \, dx = x^2/2 + C, \int x^2 \, dx = x^3/3 + C, \int e^{ax} \, dx = e^{ax}/a + C$$