

The Chemistry of Calculus: The Basic Formulas and Rules for Differentiation

Structure of Functions

Whenever you are faced with a function, determine its structure! Is it a sum, a product, a quotient, or a function inside a function? What parts are constants, either additive constants or multiplicative constants? Often it is helpful to put a function into a more 'power-full' form: $1-x^2 = x^{i-2}$, $\frac{1}{1-x^2} = (1-x^2)^{-1}$.

'ATOMS': Basic Formulas for Differentiation

Additive Constants: $\frac{d}{dx}(c) = 0$

Powers: $\frac{d}{dx}(x^n) = nx^{n-1}$ Note: powers have variable in bottom, constant in top.

Exponentials: $\frac{d}{dx}(e^x) = e^x$ Self-replicating! Note: exponentials have variable in exponent and constant in bottom.

Logarithms: $\frac{d}{dx} \ln(x) = \frac{1}{x} = x^{-1}$

Trig functions: $\frac{d}{dx} \sin(x) = \cos(x)$, $\frac{d}{dx} \cos(x) = -\sin(x)$

'Molecular Bonds': Basic Rules for Differentiation

Multiplicative Constant Rule: $\frac{d}{dx}(c \cdot f(x)) = c \frac{d}{dx} f(x)$ (Constants are your friends!)

Sum Rule: $(f + g)' = f' + g'$ The derivative of a sum is the sum of the derivatives.

Product Rule: $(f \cdot g)' = f' \cdot g + f \cdot g'$

Quotient Rule:

$$\frac{f'}{g} = \frac{f'g - fg'}{g^2}$$

Chain Rule: "a function inside a function"

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

Here u is the inside function: the guts of the function, the Unified inside.

The Chemistry of Calculus: The Basic Formulas and Rules for Integration

Structure of Functions

When integrating, you should determine the structure of the function being integrated. Is it a sum or is there a function inside a function? What parts are constants, either additive constants or multiplicative constants? If it is a product or quotient, sometimes algebra can transform the function. Often it is helpful to put a function into a more 'power-full' form: $1/(2x+3)^2 = (2x+3)^{-2}$, $1/(1-x^2) = (1-x^2)^{-1}$.

'ATOMS': Basic Formulas for Integration

Antiderivative: $\int dx = x + C$

Powers: $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ if $n \neq -1$.

When $n = -1$: $\int 1/x dx = \ln(x) + C$

Exponentials: $\int e^x dx = e^x + C$ Self-replicating!

Trig functions: $\int \sin(x) dx = -\cos(x) + C$, $\int \cos(x) dx = \sin(x) + C$

'Molecular Bonds': Basic Rules for Integration

Multiplicative Constant Rule: $\int c f(x) dx = c \int f(x) dx$ (Constants are your friends!)

Sum Rule: $\int (f + g) = \int f + \int g$ The integral of a sum is the sum of the integrals.

U-Substitution | the chain rule backwards: Choose 'u' to be the inside function (the gUts of a function) of some part of the integrand. Calculate 'du' and substitute everywhere, including the limits.

Integration by Parts | the product rule backwards:

$$\int_a^b u dv = uv \Big|_a^b - \int_a^b v du$$

Special Algebra Techniques to Transform Integrand: partial fraction decomposition, trig substitution, complex exponentials, etc.