Suggested Projects

# Finance and Economics

in class we will develop a basic Monte Carlo blue print algorithm for pricing a range of financial derivatives, including the European Call and Put, Cash-or-Nothing, Barrier-Options, and Look-back options, and compare the results with analytic results. Projects that build off of this include

1. **Pricing the *American Put***: there is no analytical formula known for this commonly traded contract. The best-known pricing method uses `Binomial Trees’. This method can be easily implemented in Excel (for a small number of subdivisions) and in R (for a large number). This project would first create an R-implementation of the binomial tree method, and then create an implementation using Monte-Carlo Methods. Questions involve speed of convergence, and accuracy, as usual, but also questions of `optimal exercise-time’ or `stopping algorithms’. Modeling the `frontier’ of optimal exercise is an open problem
2. **Alternatives to the Black-Scholes Model**:
   * **Fat Tails**: It has often been questioned whether it is true that stock returns are really normally distributed. Large scale jumps (especially downward) do seem to be more frequent than the normal distribution predicts. So how can we change the normal distribution to be more realistic? Does the the so-called implied-volatility-smile imply an alternative distribution?
   * **Stochastic Volatility**: the blue print algorithm assumes constant volatility for the duration of the contract. How would allowing the volatility to change over time influence the price? Might this explain the volatility-smile?
3. **Basket-Options**: That’s the problem of pricing options with two or more (correlated) underlyings. For example, suppose that a contract’s payoff is $100 if both underlying stocks end up higher than their respective strike prices. How much should such a contract be worth?
4. **Swaptions**: That’s the problem of pricing Option on Interest rate swaps. For example in 2005, Radnor Township wrote a swaption on its $50 million bond issue of 2004, that netted Radnor about $1 million. How was the value of that contract determined, and what are the risks Radnor took on?

# Other Ideas

1. **Random Walks on N-dimensional lattices:** Imagine a person walking from grid point to grind point, each time rolling a dice to decide where to go. Will that person ever get back to where she started? How likely will she get to a point a certain distance away? How long will it take on average?
2. **Random Walks on Graphs:** Same questions as above, except that the lattice is some graph. Often one considers cases, where the probability to go from one vertex to another varies (Markov Chains)
3. **The Traveling Salesman Problem**: Imagine you are given a list of cities to visit exactly once. You are also given the distances between all cities. What is the shortest path you can devise? This is a famous and hard problem, because as the number of cities increases, the number of possible paths grows exponentially. One way to approach it is through a Monte Carlo technique called Simulated Annealing.
4. Another MC Method for finding an optimal solution to a problem is called the **Genetic Algorithm.** It starts out with an approximate solution, then creates a new generation through a (random) mutation, either rejects that generation, or accepts it in which case it further mutates it until some stationary state is reached. this can be applied to numerous different contexts, including basketball rankings.
5. a question i thought up and don’t know the answer to: Suppose a society places a strong disapproval penalty on a man marrying a taller woman. What effect does this have on **the height distribution** in that society in successive generations? Could it be that a strong penalty results in a shorter and shorter population?
6. **Cosmology**: one can model the creation of galaxies by first creating a random distribution of points, within a square, say, then throwing in a gravitational law, then letting the points move towards the point of greatest attraction. Find out what happens.
7. **Game Theory**: some games, such as Battleship can be programmed to be played with a MC algorithm. Concepts such as the **Nash Equilibrium** can be explored with MC methods.